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A. I. Tolstykh

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16. Abstract

Description of a method of obtaining numerical solutions to the Navier-Stokes equations for various problems of aerodynamics in a wide range of Reynolds numbers. The proposed method is based on "stretching" regions in which the unknown functions have large gradients. Problems of the flow in a shockwave or flows past blunt bodies at fairly large Reynolds numbers are taken as examples where the proposed method is applicable. In the case of the latter type of flow, a difference scheme with a high order of accuracy is presented.

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ONE METHOD OF NUMERICAL SOLUTION OF THE NAVIER-STOKES EQUATIONS FOR A COMPRESSIBLE GAS

A. I. Tolstykh

1. The necessity of using the Navier-Stokes equations frequently arises when solving many problems of aerodynamics. This is done either for low Reynolds numbers (for example, in the case of low density gas flow) or at large Reynolds numbers, when boundary layer theory does not provide an effective description of the flow in the entire region being considered. The latter class of problems includes the problem of the interaction of a shockwave with a boundary layer, the wake behind a body, etc. /78*

Even in those cases when the existence of stable solutions of the Navier-Stokes equations may be assumed, great difficulties are encountered in obtaining them by means of finite difference methods. These difficulties are related, in particular, to the large gradients of the unknown functions; these gradients are substantial in the regions of the boundary layer and the shockwaves. Under these conditions, the use of difference schemes of low order of accuracy leads to the fact that, in the case of real values of the difference grid steps, the errors of approximating the differential equations may be comparable with the terms of the difference equations containing the viscosity coefficient.

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*Numbers in the margin indicate the pagination of the original foreign text.

The effect of using schemes of high order of accuracy is greatly reduced, in the first place, due to the large values of the higher derivatives of the unknown functions included in the expression for the scheme approximation error. In the second place, it is reduced due to the frequently occurring non-monotonic nature of the solutions of the difference equations, which becomes very noticeable at large values of the Re and M numbers.

The use of existing difference schemes for complete equations for the flow of a viscous compressible gas [1 — 4] /79
(in addition to those enumerated in [1 — 2], we should also note [3] and [4]), is apparently limited by the region of comparatively small values of the Re numbers. A description is given below of a method for obtaining numerical solutions of the Navier-Stokes equations for different problems of aerodynamics in a wide range of Re numbers. Examples are given of calculations to determine the effectiveness of the method in relatively simple cases.

2. In the flow region being considered, let us assume there are zones of large gradients of the unknown functions (for example, shockwaves or boundary layers), which impede the effective use of the difference schemes.

For purposes of simplicity investigating a plane or axisymmetric case, let us select an orthogonal system of coordinates s, n such that the discontinuity lines, which these zones cross at $\rightarrow \infty$, intersect the lines s = const at non-zero angles.

In the general case, let us assume that it is necessary to find the solution of the boundary value problem for the Navier-Stokes equations in the region

$$0 \leqslant s < s_0, \quad n_0(s) \leqslant n \leqslant n_b(s) \leqslant \infty, \quad 0 \leqslant t \leqslant T,$$

where t is the time, $n_0(s)$ — body surface, and $n_b(s)$ — a certain "external" boundary. Instead of the variables s, n, let us introduce new independent variables $\bar{s}$, ζ

$$\bar{s} = s; \quad \zeta = \zeta(u, v, s, n), \tag{1}$$

where u and v are the velocity components corresponding to the coordinates s and n.

We shall require that the function ζ satisfy the following conditions:

$$\zeta|_{n=n_0} = 0; \quad \zeta|_{n=n_b} = 1;$$
$$\gamma = \partial\zeta/\partial n > 0 \quad (n_0 \leqslant n \leqslant n_b).$$

The first condition is the norming condition, the second condition indicates that the transformation is one-to-one.

In addition, let us select the function ζ so that it changes by one order of magnitude in regions with large gradients, i.e., so that it approximately repeats the change along the coordinates of the velocity components which are undergo a discontinuity at $Re \to \infty$.

It may be expected that the solution of the original differential equations, described in the coordinates (1), will contain a comparatively slowly changing function of the variable ζ, so that on the plane $\bar{s}$, ζ the regions disappear with small characteristic dimensions striving to zero with an increase in the Re number. In this case, the errors arising during the substitution of the differential equations by difference equations, due to the comparatively small values of the higher derivatives with respect to ζ of the unknown functions, must be greatly reduced as compared with similar errors in the case of approximation in the physical s, n plane. It may also be assumed that there is a decrease in the non-monotonic nature of the solutions of difference equations in the case of schemes of higher order of accuracy.

In contrast to the regular bunching of nodes of a difference grid, we should note that in regions with large gradients of the functions (for example, [5]), the transformation (1) does not /80 require an a priori knowledge of the distribution of these regions. In the case of a uniform grid, along the coordinate ζ this bunching is performed "automatically" in the process of obtaining a solution.

After the change to the coordinates (1) in the initial equations there is a new function $\chi(s, \zeta) = \partial\zeta/\partial n$. The second of the equations (1), which is a definition of the function ζ, closes the system. The variable n, included in (1) and in the Lamé coefficients for a curvilinear system of the coordinates s, n, may be determined with sufficient accuracy by means of the quadrature

$$n(\zeta, s_0) = \int_0^{\zeta} \frac{d\zeta}{\chi(s_0, \zeta)} \qquad (2)$$

3. Let us establish the effectiveness of introducing the transformation (1) in two characteristic cases, when there is either a shockwave or a boundary layer in the field of steady flow.

The problem of the structure of a rectilinear shockwave is an example, when the selection of the function ζ is particularly simple. We should note that this classical problem is frequently a criterion for the applicability of a certain difference scheme (for example [6]).

Let us assume all the functions depend only on one coordinate n and at $n \to \infty$ the flow is supersonic flow. Relating the velocity v and the density ρ to the velocity V_∞ and density ρ_∞ of an unperturbed flow, and the pressure p and the enthalpy h to the values $\rho_\infty V^2$ and V^2, we may write the boundary conditions of the problem in the form

$$\left.\begin{array}{l} \rho = 1;\quad h = h_\infty = \dfrac{1}{(\gamma-1)M_\infty^2};\quad v = v_\infty = -1 \text{ for } n \to +\infty; \\ h = \dfrac{1}{2} + \dfrac{1}{(\gamma-1)M_\infty^2};\quad v = v_0 = -\left(\dfrac{\gamma-1}{\gamma+1} + \dfrac{2}{(\gamma+1)M_\infty^2}\right) \\ \qquad\text{for } n \to -\infty, \end{array}\right\} \tag{3}$$

where M is the M number of the unperturbed flow; γ — adiabatic index.

Setting $\zeta = (v - v_0)(v_\infty - v_0)$, we find that the function ζ satisfies all of the requirements formulated above. The equations of the gas flow in a one-dimensional shockwave may be written in the form

$$\left.\begin{array}{c} \dfrac{d\rho v}{d\zeta} = 0;\quad \rho v(v - v_0) + \dfrac{\partial p}{\partial \zeta} = \dfrac{4}{3}(v - v_0)\dfrac{d\mu\chi}{d\zeta}; \\ \rho v \dfrac{d}{d\zeta}\left(h + \dfrac{v^2}{2}\right) = \dfrac{1}{\Pr}\dfrac{d}{d\zeta}\left(\mu\chi\dfrac{dh}{d\zeta}\right) + \dfrac{4}{3}(v - v_0)\dfrac{d}{d\zeta}(\mu\chi v); \\ \chi = \dfrac{d\zeta}{dn}\dfrac{\mu_*}{\rho_\infty V_\infty};\quad p = \dfrac{\gamma-1}{\gamma}\rho h;\quad \mu = h^{\omega}, \end{array}\right\} \tag{4}$$

where Pr is the Prandtl number; ω — the constant ($0.5 \leq \omega \leq 1$); μ — viscosity coefficient pertaining to the viscosity coefficient μ_* when h = 1.

Let us introduce the difference grid $\zeta_i = i\Delta\zeta$ $(i = 0, 1, \ldots, 1/\Delta\zeta)$ and let us write for Equations (4) an admittedly rough difference /81 scheme of the first order of approximation. For this purpose, let us replace the first derivatives of all the functions, except for pressure, by unilateral difference relationships. We shall approximate the derivatives $dp/d\zeta$ by means of the integral differences. We may write the approximation of the derivatives $[d(\mu\chi\, dh/d\zeta)]/d\zeta$ with an accuracy up to terms on the order of $O(\Delta\zeta)$, without centering the coefficient $\mu\chi$ with respect to the nodes, at which the derivative $dh/d\zeta$ is calculated. The system of difference equations for the system (4) with the boundary conditions (3) and the apparent condition $\chi(1) = 0$ may be solved by the method of successive approximations by introducing the relaxation parameter. Each equation is regarded as a linear equation during one iteration.

The calculations were carried out for the case $M_\infty = 10$, Pr = 0.75, $\gamma = 1.4$, and $\omega = 0.5$. Figure 1 shows the relationships of pressure, enthalpy, and the value of χ * for a different number of nodes in the grid (N = 5, 10, and 20). The gasdynamic functions have comparatively*, so that any difference approximation of the inertial terms must not lead to great errors. This is confirmed by the fact that the curves for the functions h and p, even in an unfavorable case of a rough scheme when N = 5, 10, and 20, differ from each other very little. The small difference in the curve of the function χ may be explained by the first order of approximation for the term with

*[Translator's Note: Illegible in foreign text.]

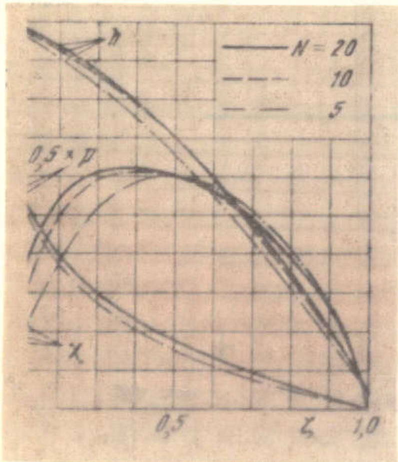

Figure 1.

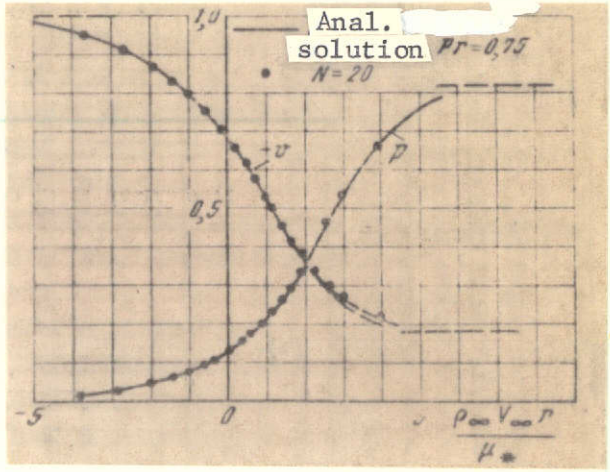

Figure 2.

the viscosity coefficient in the second equation (4).

To change to the physical plane, let us place the origin at the point $\gamma = 1/2$ and we shall calculate the integrals

$$n_i = -\int_{0.5}^{\zeta_i} \frac{d\xi}{\chi(\xi)} \quad [i = 1, 2, \ldots, (N-1)]. \tag{5}$$

In view of the fact that the function χ has zeros when $\zeta = 0$ and 1, to derive (5) special quadrature formulas are formulated. The results of the integration are given in Figure 3, which shows the velocity and pressure profiles (points) together with the accurate solution (solid line) corresponding /82 to the case $Pr = 0.75$, $\omega = 1/2$, and $M_\infty = 10$.

4. Let us now consider the case of axisymmetric flow, in which at large Re numbers there is a boundary layer. This case is advantageous due to the fact that the results obtained may be compared with known data for nonviscous flow and a boundary layer.

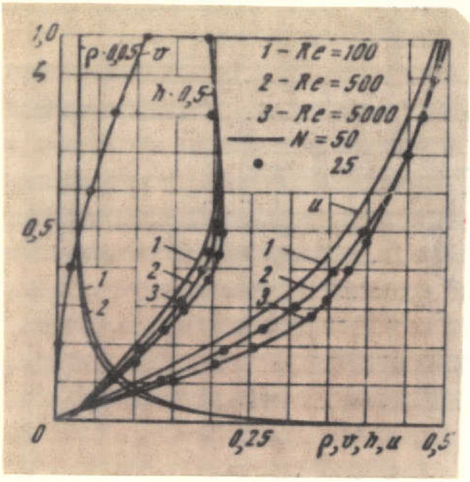

Figure 3.

Let us select the system of coordinates s, n connected with a sufficiently smooth contour of the body, placing its origin at a critical point and directing the n axis along the normal to the surface. We shall try to find a numerical solution for the complete steady-state equations of Navier-Stokes in the $0 \leqslant n < n_b(s)$, $0 \leqslant s \leqslant s_0$, band, where $n_b(s)$ is the front of the outgoing shockwave, which we shall assume is a discontinuity surface. The conditions of capture and the condition for the temperature when n = 0, the symmetry conditions when s = 0, and the Hugoniot relationships when $n = n_b(s)$ will be used as the boundary conditions. Strictly speaking, the conditions on the shockwave are insufficient in the case of a viscous gas. However, when writing the difference scheme, we shall always approximate the equations on the $n = n_b$ line by means of the boundary and inner nodes of the region, formally obtaining a closed system. The basis for this is as follows: in the first place, for our purposes, the inaccuracies in the boundary conditions are insignificant. In the second place, at large Re numbers, the flow around a wave is practically nonviscous, so that the method of approximating the terms with the viscosity coefficient in the vicinity of the line $n = n_b$ must not have a great influence upon the solution obtained. We will impose no additional conditions on the line $s = s_0$. With a sufficiently large value of s_0, the perturbations which are propagated upward along the flow from the $s = s_0$ line must be rapidly damped.

Let us introduce the function ζ as follows:

$$\zeta = \frac{(u/u_b) + Cn}{1 + Cn_b} \tag{6}$$

where C is a constant which is selected sufficiently large so as to guarantee the monotonic nature of the function $\zeta(n)$.

A suitable selection of C makes it possible to use the transformation (5) for a wide class of problems (flow in the separation zone, in the wake, the nozzle, etc.).

In the Navier-Stokes system described in the s, n coordinates [7], we shall change to the independent variables $\bar{s}$, ζ. For a numerical solution of the equations obtained, we may use the well known difference schemes. However, the calculations were carried out by means of a special scheme having a high order of approximation with respect to ζ. We shall describe this scheme in general terms.

The equations for the normal components of momentum, energy, and continuity may be represented in the following form: /83

$$\left.\begin{aligned}
&\text{a)}\quad W(\rho v) + \frac{\partial}{\partial \zeta}\left[\rho v(v - Gu) + \frac{\gamma - 1}{\gamma}\rho h - \tau_{nn}\right] - \frac{k\rho u^2}{H_\chi} = \frac{1}{\mathrm{Re}} T_1;\\
&\text{b)}\quad W(\rho E) + \frac{\partial}{\partial \zeta}[\rho E(v - Gu) - q_n] = \frac{1}{\mathrm{Re}} T_2;\\
&\text{c)}\quad W\rho + \frac{\partial}{\partial \zeta}\rho(v - Gu) = 0.
\end{aligned}\right\} \tag{7}$$

Here W is the differential operator containing only convective terms of the equations and not containing derivatives with respect to k — the curvature of the contour; H = 1 + kn:

$$E = h + \frac{u^2 + v^2}{2}; \quad G = \frac{\partial n(\bar{s}, \zeta)/\partial \bar{s}}{H};$$

where τ_{nn} and q_n are, respectively, the stress tensor components and the normal component of the thermal flux; Re — Reynolds number calculated according to the parameters of the unperturbed flow, viscosity at * = 1, and radius of curvature for the contour at the critical point R_0. All linear dimensions pertain to R_0, and the curvature refers to $1/R_0$; T_1 and T_2 are the remaining terms of the equations, containing the viscosity coefficient.

On the difference grid $\bar{s}_i = i\Delta s, \; \zeta_j = j\Delta\zeta \; (i = 0, 1 \ldots s_0/\Delta s, \; j = 0, 1, \ldots, 1/\Delta\zeta)$, we may introduce the operators A_+ and A_- which are in operation according to the formulas

$$\left.\begin{aligned} (A_+ f)_{ij+1/2} &= \frac{1}{12}(5f_{ij} + 8f_{ij+1} - f_{ij+2}); \\ (A_- f)_{ij+1/2} &= \frac{1}{12}(5f_{ij+1} + 8f_{ij} - f_{ij-1}). \end{aligned}\right| \qquad (8)$$

We may write the difference scheme for Equation (7) in the following form:

$$\left.\begin{aligned} [(\rho\tilde{v})_{ij+1} - (\rho\tilde{v})_{ij}]/\Delta\zeta + (A_+ W\rho)_{ij+1/2} = 0; \\ \tilde{v} = v - Gu. \end{aligned}\right| \qquad (9)$$

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approximating the derivatives with respect to $\bar{s}$, included in the operator, according to a three-point scheme by using the values of the functions on the previous lines $\bar{s}_i = \text{const}\ (i \geqslant 2)$. It may be readily established that the approximation error at the point (i, j + 1/2) has the form

$$\frac{(\Delta\zeta)^2}{24}\frac{\partial^2}{\partial\zeta^2}\left(\frac{\partial\omega v}{\partial\zeta} + W\rho\right)_{ij+1/2} + O(\Delta\zeta^3 + \Delta s^2),$$

and that the scheme (9) at the point (i, j + 1/2) approximates the equation (8) and its solutions with an accuracy of $O(\Delta\zeta^3 + \Delta s^2)$.

Applying the operators A_+ and A_- to the expressions

$$W\rho v - \rho\frac{ku^2}{H\zeta},\ W(\rho E),$$

we obtain similar schemes for the equations (7a, b). Thus, the error of the approximation, when writing the terms in the right sides of (7a, b), with the second order of accuracy with respect to all the variables, will have the form /84

$$\max\left[O(\Delta\zeta^3 + \Delta s^2),\ O(\Delta\zeta^2\,\mathrm{Re} + \Delta s^2)\right].$$

It is apparent that, in the case of large Reynolds numbers, the approximating properties of the scheme being considered do not become worse with a change to the equations (7a, b), at least in the region of small gradients of the desired functions.

Noting that the operators A_+ and A_- determine the quadrature formulas of the form

$$\int_{\zeta_j}^{\zeta_{j+1}} f\,d\zeta = (A_{\mp}f)_{j+1/2}\,\Delta\zeta + O(\Delta\zeta^4),$$

it may be readily established that, with a corresponding approximation of the terms containing the viscosity coefficient, the schemes for Equations (7a, b) are conservative and may be obtained by means of the laws of conservation for meshes of the form

$$\bar{s}_{i-1/2} \leqslant \bar{s} \leqslant \bar{s}_{i+1/2}, \quad \zeta_j \leqslant \zeta \leqslant \zeta_{j+1}.$$

The selection of the operators A_+ or A_- was determined from the condition that, in the first place, the nodes employed did not exceed the boundary of the region and, in the second place, that in the linear approximation all of the eigennumbers of the corresponding operators for the change with respect to the modulus do not exceed unity. Satisfaction of the second condition was related to the magnitude and direction of the velocities.

The equation of momentum, in projection on the $\bar{s}$ axis, after certain transformations and substitutions of the derivatives by finite differences, was written for the line $\bar{s} = \bar{s}_i$ in the following form

$$\frac{1}{\mathrm{Re}} \langle \partial \mu \chi / \partial \zeta \rangle_{ij} = a_{ij} + \frac{b_{ij}}{\chi_{ij}} \quad (j = 0, 1, \ldots, 1/\Delta\zeta), \tag{10}$$

where a_{ij} and b_{ij} do not contain the values of the function χ on the i^{th} line, and $\langle \partial \mu \chi / \partial \zeta \rangle_{ij}$ is the difference approximation of the derivative $\partial \mu \chi / \partial \zeta$. We reach an equation of the form (11), using the laws of conservation of tangential momentum for an elementary mesh.

For the function χ, Equation (10) plays an important role in the system obtained. In particular, it may be shown that, depending on the signs of the coefficients a_{ij} and b_{ij}, the function χ may correspond either to separated or non-separated flow.

The general solution of the system of algebraic equations by an iteration method was as follows. During one iteration, there is a sequential transition from the $i^{\underline{th}}$ line to the $(i + 1)^{\underline{th}}$ line, according to an implicit scheme, and in the case $i \geqslant 2$ the values found at two preceding lines were used. The values of the functions when $\bar{s} \geqslant s_{i+2}$, and also certain other terms of the equations (7a — c) were selected from the previous iteration. The specific characteristics of solving the equations along each line consisted of the fact that the rate v_{ij} was determined, not from the equation (7a), but from the continuity equation (7c). On the other hand, the density ρ_{ij} was found from the equation for a normal momentum (7a).

A special method was used to determine k_{ij} from Equation (7b). The solution of Equation (10), after replacing the derivative in the left side by a two-point scheme, was reduced to the subsequent calculation of the values χ at the points j = 1, 2, ..., $1/\Delta\zeta$. The boundary values χ_{i0} were determined from the condition that the velocity $v_{i,1/\Delta\zeta}$, found from Equation (9), /85
equal the velocity at the discontinuity v_b. The process of determining χ_{i0} employed the Newton method, and the required convergence was obtained as a result of one-two iterations.

The values found for k_{ij} on the line $\bar{s} = s_i (i = 0, 1, \ldots, s_0/\Delta s)$ were used to determine the coordinate n_{ij} of the nodes in a physical plane, and then the values of the tangential velocity u_{ij} were calculated from the finite relationships of the form (6).

After completing a regular iteration, i.e., determining the parameters on all the lines $\bar{s} = s_i \; (i = 0, 1, \ldots, s_0/\Delta s)$, the angles of inclination of the shockwave were calculated and, consequently, the values of all the functions for the wave for the subsequent iteration. In final form, any of the values obtained for f^m_{ij} may be written in the form

$$\bar{f}^m_{ij} = \lambda f^m_{ij} + (1-\lambda) f^{m-1}_{ij},$$

where m is the iteration number, λ — relaxation parameter. Even without an optimum selection of the parameter λ (in all the calculations it was assumed that $\lambda \approx 0.1$), the convergence up to 3 — 4 signs, as a rule, was achieved after 200 — 300 iterations in the case of a zero approximation by means of the constant and linear functions of ζ.

5. Let us discuss the results of calculations carried out for the numbers $M_\infty = 10$, $\gamma = 1.4$, and Pr = 0.72 for the cases of a condensed sphere $h_w = h_\infty = 1/(\gamma - 1) M_\infty^2$ и $\omega = 0.5$] and Pr = 0.72. With the number of the grid nodes

$$N = \frac{1}{\Delta\zeta} \leqslant 50, \quad M = \frac{s_0}{\Delta s} \quad 8$$

the value of $s_0 \leq 1.2$ was varied and had no noticeable influence upon the data.

Figure 3 shows the profiles of velocity u and v, enthalpy h, pressure p, and density ρ on the line $\bar{s} = 0.6$ at Re = 100, 500, and 5000, corresponding to Reynolds numbers calculated according to the parameters of unperturbed flow, approximately equalling 640, 2300, and 32,000, respectively. For the cases Re = 500 and 5000, Figure 4 gives data obtained for N = 25 (dots) and N = 50 (solid lines). These results barely differ. Apparently, a sufficient computational accuracy may be obtained by means of a relatively large step $\Delta\zeta$ of the difference grid. We should also

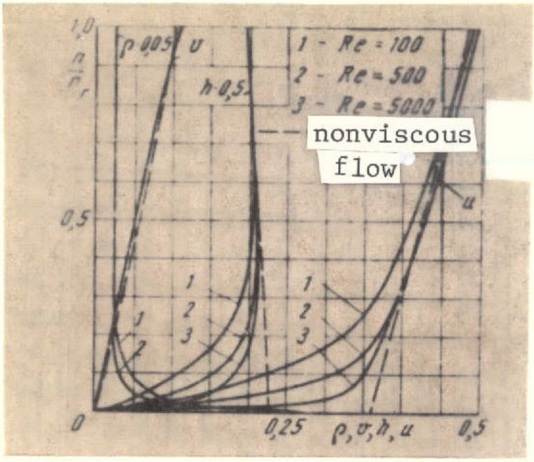

Figure 4.

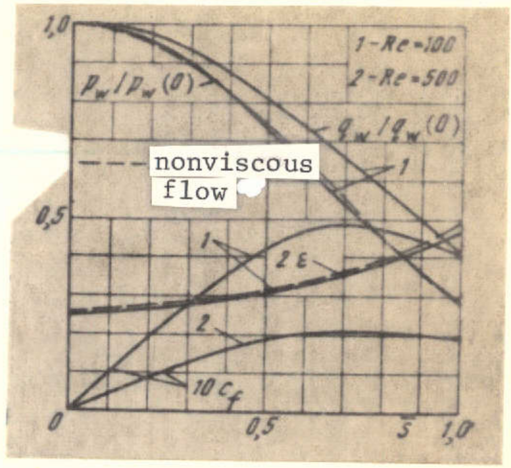

Figure 5.

note that, with an increase in the Re number, the dependences of the gasdynamic parameters on the ζ coordinate are comparatively conservative and do not have sharply expressed gradients. The basic changes occurring as the boundary layer becomes thinner in the physical plane, are concentrated at the function $\chi(\bar{s}, \zeta)$ characterizing the vorticity of the flow. Close to being constant in the "external" region, this function begins to increase rapidly, assuming larger values on the body surface, the larger is the Reynolds number. However, the width of the region corresponding to the boundary layer, is finite in the $\bar{s}$, ζ plane and does not vanish when Re $\to \infty$.

The changes in the gasdynamic parameters along this line * = 0.6 in the physical plane are given in Figure 4. For purposes of comparison, this figure gives the results of calculating nonviscous flow [8] $M_\infty = 10$, $\gamma = 1.4$ corresponding to the cross section s = const.

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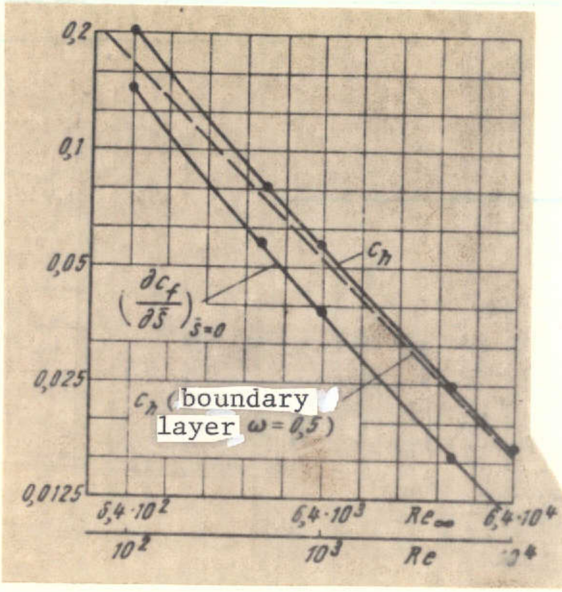

Figure 6.

As may be seen in Figure 4, the profiles of the gasdynamic functions obtained by means of the Navier-Stokes equations, except for the boundary region, are close to the flow profiles of a nonviscous gas. The basic changes of these functions take place in a narrow zone around the wall where, for example, the density changes by approximately a factor of 20. /86

Figure 5 gives the distribution along the body surface of the friction coefficient $c_f = (\mu/Re)(\chi\partial u/\partial\zeta)|_{\zeta=0}/\rho_\infty V_\infty^2$, the thermal flux $q_w = (\mu/Re\,Pr)(\chi\partial h/\partial\zeta)|_{\zeta=0}/\rho_\infty V_\infty^3$, the pressure $p_w = p(\bar{s}, 0)$, and also the magnitude of the shockwave departure $\varepsilon = n_b$.

The dashed lines give the dependence $p_w(\bar{s})$ and $\varepsilon = n_b(\bar{s})$ for the case of ideal flow. The smaller value of the distance of the shockwave departure for a viscous gas corresponds to a negative value of the thickness of the boundary layer displacement in the case being considered of a strongly cooled surface ($h_w = h_\infty = 0.025$). Finally, in logarithmic scale, Figure 6 gives a change in the derivative $(dc_f/d\bar{s})_{\bar{s}=0}$ and the heat transfer coefficient $c_h = q_w(0)/(h_0 - h_w)$, where h_0 is the braking enthalpy, with a change in the Re number. The linear nature of these functions at $Re > 10^3$ and the angle of inclination of the lines point to an inversely proportional dependence of the friction coefficients and heat transfer coefficients on $\sqrt{Re}$. For purposes of comparison, Figure 6 shows the change in the coefficient c_k with the Reynolds number, according to the boundary layer theory [9] when $\omega = 0.5$. /87

Thus, the results of the calculations, performed on the basis of the complete Navier-Stokes equations, closely coincide at large Reynolds numbers with the data for the corresponding nonviscous flow and boundary layer.

In conclusion, we would like to note that the transformation (1) may be simplified for the case which is not considered here, when it is advantageous "to stretch" the region both of the boundary layer and of the shockwave.

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